LITHOFLEX WORKSHOP

24-25 JUNE 2008

StatoilHydro Research Centre Rotvoll – Trondheim – Norway

Lithoflex theoretical background:

Part V Regional flexure modelling

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Topics

- Regional flexure modelling
  - Full plate and broken plate model
- Forward flexure calculations
  - Te constant and variable
- Inverse flexure calculations
  - Necessary constraints:

    crustal thickness

    equivalent load

    relative importance of internal loads and topographic loads

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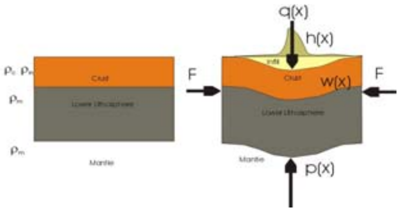
Regional equilibrium (Flexural Isostasy)

Model of the flexure of a thin plate



$$D\frac{d^4w(x)}{dx^4}+F\frac{d^2w(x)}{dx^2}+(p(x)-q(x))=0$$

$$p(x)=g\,w(x)\,\rho_m$$

$$q(x)=g\,(w(x)\,\rho_{in}+h(x)\,\rho_l\,)$$

q = downward force

p = upward force

$w(x)$ = flexure

$h(x)$ = topography

ρ_m = mantle density

ρ_{in} = infill density

ρ_l = load density

D = flexural rigidity

F = compressional force

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Regional equilibrium (Flexural Isostasy)

Flexural rigidity:

$$D=\frac{ET_e^3}{12(1-\sigma^2)}$$

T_e = elastic thickness

E = Young Modulus

σ = Poisson ratio

Typical values:

E= 10^{11} N/m^2

σ = 0.25

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Typical values for elastic thickness

- Cratonic interiors: 60 to > 100km
  - Greater than crustal thickness. Mantle contributes to rigidity
- Rift areas : (Afar Rift)near to 10 km
- Old lithosphere with decoupled mantle (Carpathians, Tien Shan): 20-40 km
- Oceanic crust: Te increases with thermal age at time of loading

Perez-Gussinye et al. 2004; Watts 2001

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Special cases-solutions

- Loading of rectangular plate supported at one edge ("broken plate")
- Continuous plate loaded by line-load (chain of volcanoes, linear orogen)
  - Analytical solutions exist
- Many other special solutions exist- engineering literature.
- Handycap: applies to specific situation - poor flexibility

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Line load on plate

- Continuous plate

$$w(x) = \frac{P_0 \lambda}{2(\rho_m - \rho_{in})g} e^{-\lambda x}(\cos \lambda x + \sin \lambda x)$$

$$\lambda = \left[\frac{(\rho_m - \rho_{in})g}{4D} \right]^{1/4}$$

P_0 = line load applied in x = 0

ρ_m = mantle density

ρ_{in} = infill density

g = 9.81 m/s^2

D = flexural rigidity

- Broken plate

$$w(x) = \frac{2 P_0 \lambda}{(\rho_m - \rho_{in})g} e^{-\lambda x}(\cos \lambda x)$$

$$\lambda = \left[\frac{(\rho_m - \rho_{in})g}{4D} \right]^{1/4}$$

P_0 = line load applied in x = 0

ρ_m = mantle density

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g = 9.81 m/s^2

D = flexural rigidity

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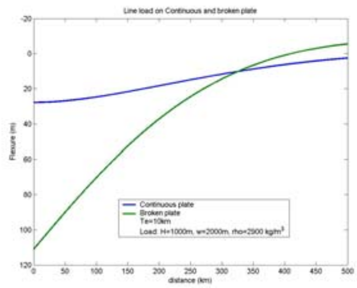

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General solution in frequency domain
(neglect compressional forces)

$$W(\vec{k}) = F(\vec{k})H(\vec{k}) = \frac{\rho_l}{\rho_m - \rho_{in} + \frac{D}{g}|\vec{k}|^4} H(\vec{k})$$

Flexural rigidity $\boxed{D = \frac{ET_e^3}{12(1-\sigma^2)}}$

T_e = elastic thickness
E = Young Modulus
σ = Ratio Poisson

Typical values:
E= 10^{11} N/m^2
σ = 0.25

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Transition to local compensation:

With very low flexural rigidity or for very small wave numbers (long wavelengths) the regional isostasy goes over into local Airy type compensation:

$$\boxed{\begin{array}{l} D \to 0 \ \textit{and / or} \ \ k \to 0 \\ W(k) = \dfrac{\rho_c}{(\rho_m - \rho_c)} H(k) \end{array}}$$

With very high flexural rigidity or for very great wave numbers (short wavelengths) the loading does not deform the plate.

$$\boxed{\begin{array}{l} D \to \infty \ \textit{and / or} \ \ k \to \infty \\ W(k) = 0 \end{array}}$$

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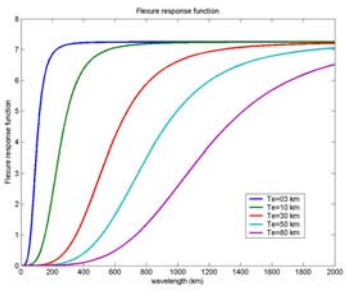
Flexure response function



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Calculation methods of flexure

- spectral domain:
  $\boxed{W(\vec{k}) = H(\vec{k})T(\vec{k})}$
- space domain (convolution):
  $\boxed{w(\vec{x}) = FT^{-1}[W] = FT^{-1}[H] * FT^{-1}[T] = h(\vec{x}) * t(\vec{x})}$
  $h(\vec{x}) =$ Impulse response

h(x): characteristic of flexural rigidity
ASEP (Wienecke, 2006): analytical approximation

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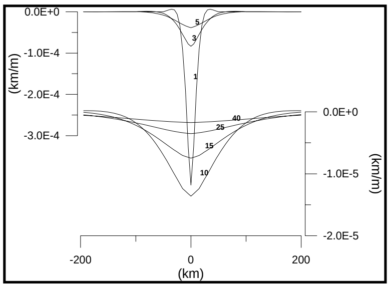
Flexure rensponse to topographic pointload

Te=1,3,5,10,15,25,40 km



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Piecewise constant flexural rigidity

- space domain (convolution):

$$w(\vec{x}) = \sum_i w_i(\vec{x}) \qquad \text{flexure}$$

$$w_i(\vec{x}) = h_i(\vec{x}) * t(\vec{x}) \qquad \text{for } x_1^i \le x \le x_2^i, y_1^i \le y \le y_2^i$$

$$w_i(\vec{x}) = 0 \qquad \text{otherwise}$$

$$h_i(\vec{x}) \quad \text{impulse response for } D_i$$

$$A_i : \text{area defined by } x_1^i \le x \le x_2^i, y_1^i \le y \le y_2^i$$

$$A_i \text{ area over which flexural rigidity assumed constant}$$

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Flexural model situations 1/2

1) Known:

load h(x), Flexural rigidity D assumed, Gravity field

To be calculated: flexure w(x)

-> Calculate flexure and corresponding gravity field.

-> fit observed gravity field.

-> Make conclusions on internal loads h(x) and D

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Flexural model situations 2/2

2) Known:

load h(x), Moho deflection w(x), Gravity field

To be calculated: flexural rigidity D

-> fit Moho deflection

-> Invert variations of flexural rigidity D

-> Convolution method allows best spatial resolution of D

-> numerically stable also in case h(x) is small

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Other methods to calculate the flexural rigidity

- Admittance of observed gravity and topography
- Coherence of observed gravity and topography

Numerical forward modelling: Finite element methods

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Admittance and Coherence

- Coherence:

$$\gamma^2_{obs}(k) = \frac{\left|H(k)B(k)^*\right|^2}{\left|H(k)\right|^2 \left|B(k)\right|^2} \quad \textit{averaged over rings of equal } k$$

- Admittance:

$$Z_{obs}(k) = \frac{H(k)B(k)^*}{\left|H(k)\right|^2} \quad \textit{averaged over rings of equal } k$$

H(k)= FT of topography;
B(k)=FT of Bouguer gravity or Gravity anomaly

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Obtaining Elastic thickness

- modeling observed admittance or coherence curves
  - Assumptions: ratio of surface and subsurface loading
  - Methods on deconvolution of subsurface masses:
  - Forsyth (1984)
  - Lowry and Smith (1994)

Subsurface masses: contribution to load and direct contribution to gravity

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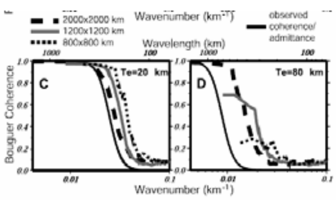
Coherence Synthetic Example



Theoretical (black line) and observed (gray and dashed black lines) (c, d) coherence functions for Te = 20 km (c) and 80 km (d). Observed coherence functions were calculated using windows of synthetic topography and gravity data of 2000 x 2000 km, 1200 x 1200 km, and 800 x 800 km.

Perez-Gussinye et al. 2004

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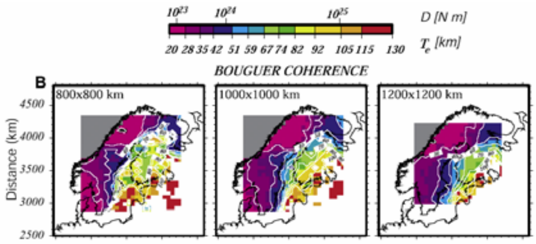
Results from Bouguer coherence



Perez-Gussinye et al. 2004

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Problems with spectral approach of comparing topography and gravity

- Large windows are necessary with joint gravity and topography
- Data must be known or interpolated to square windows
- Low-energy topography difficult to handle numerically (numerator)
- Subsurface masses are complicated to take into account for-> statistical methods

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Problems with spectral approach of comparing topography and gravity

- Analysis of topography and gravity does not approach the problem directly:
  - Topography and flexure is the direct problem. Spectral analysis of Moho and loads is the direct approach.
  - Topography and gravity mixes the effect of internal loads on loading and on the direct effect of gravity

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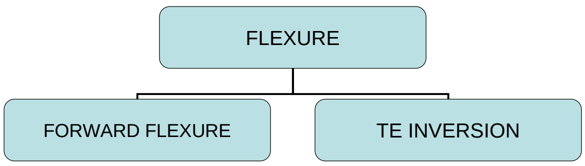
LithoFlex FLEXURE TOOL

- Goal: Regional Isostasy for crustal modelling
- Methods: Forward modelling using two methods (see next slide: convolution and spectral method)
- *LithoFlex* Tool→/FLEXURE, one can choose between two tools:



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FORWARD FLEXURE

- FIXED ELASTIC THICKNESS
  - **Fast** use the **spectral method (Spectral domain)**
  - **Precise** use the **convolution method (Spatial domain)**
- VARIABLE ELASTIC THICKNESS
  - **Precise** use the **convolution method (Spatial domain)**

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Forward calculation

- Forward flexure calculation
  - Continental area (West Siberian Basin)
  - Oceanic area (South China Sea)
- Needed:
  - Load
  - Constant Flexural rigidity or Te
  - Spatially variable flexural rigidity or Te
    - In case there are hints for variation of D or Te

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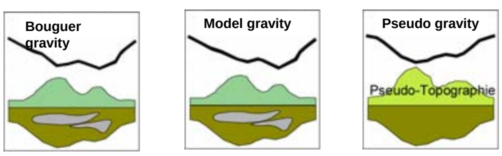
Equivalent topography

In addition to topographic masses, masses within the crust, subsurface masses have to be considered.



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Equivalent topography

- Needed: total load:
  - topographic load summed to equivalent topography
  - Equivalent topography: multiplied by crustal density equivals to intracrustal loads
  - These are due to:
    - ocean, sediments, density anomalies in crust

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Total Load: topographic and buried load

Buried load: $$L_{buried} = \sum_{i=1}^{N} h_i \rho_i - \sum_{i=1}^{N} h_i \rho_c$$

L_{buried} inner-crustal loads, h_i thickness of the i-th layer, ρ_i density of the i-th layer and ρ_c the density of the reference crust.

Equivalent topography: buried load divided by reference density:

$$h_{equivalent} = \frac{L_{buried}}{\rho_c}$$

Total load = (topography+equivalent topography) * ρ_c

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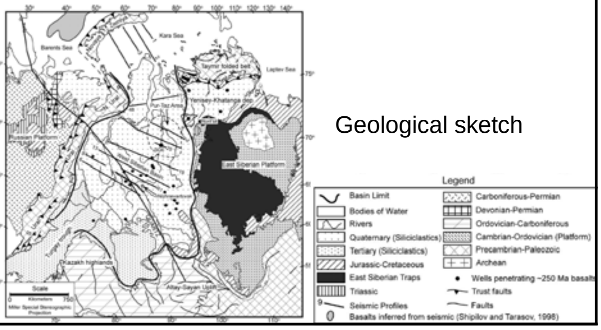
Working Area: West Siberian Basin (WSB)



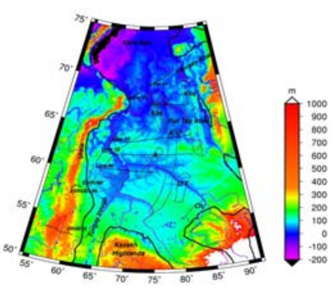
Geological sketch

Topo

Extended grid

2 070 000 m sidelength (X)
2 300 000 m widelength (Y)

Xmin=30 000 m
Xmax=2 090 000 m
Ymin=0 m
Ymax=2 300 000 m



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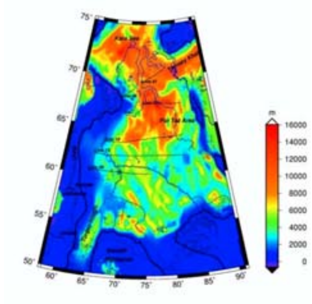
Sediment thickness



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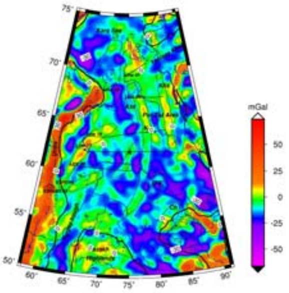
Gravity anomaly



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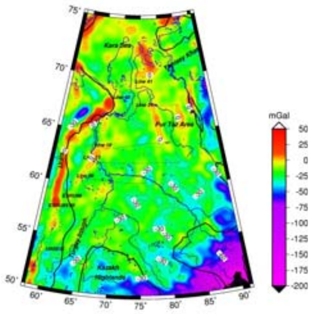
Bouguer field



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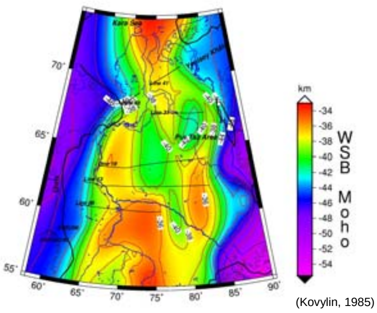
Moho



(Kovylin, 1985)

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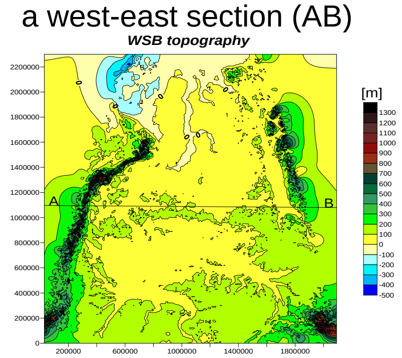
Example profile along a west-east section (AB)



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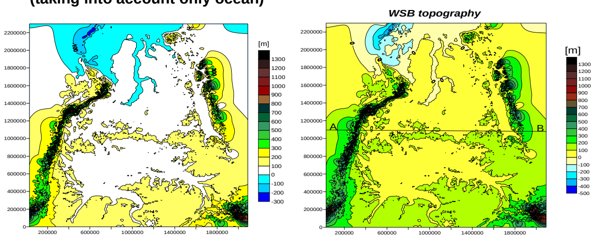
Equivalent topography

Equivalent topography
(taking into account only ocean)

topography



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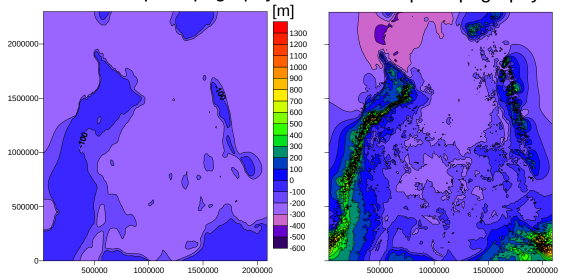
Equivalent topography of sediments

Sediment Equi-Topography

Total Equi-Topography



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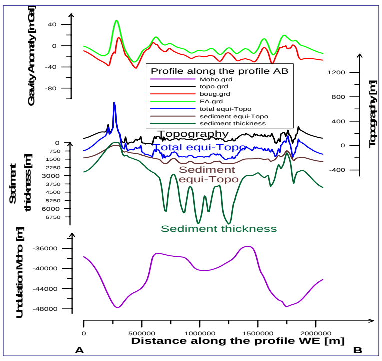
Different quantities along the WE profile (AB)



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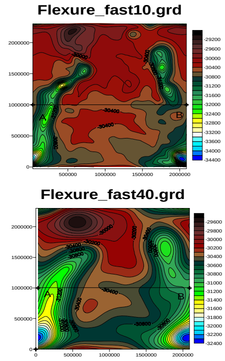
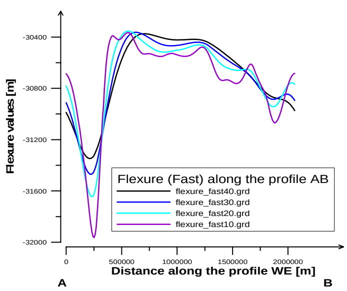
Flexure (fast)

Flexure_fast10.grd

Flexure_fast40.grd





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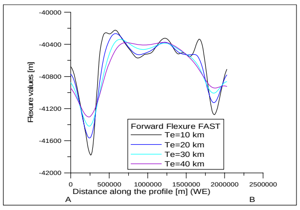
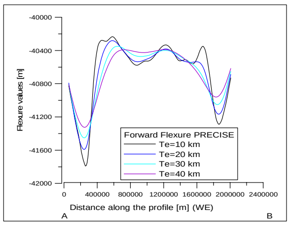
Comparison between the spectral and convolution methods





Effect of Te variation

- What parameters stiffen the plate:
  - Increase of flexural rigidity D
  - Increase of effective elastic thickness

$$D = \frac{ET_e^3}{12\left(1-\sigma^2\right)}$$

- Effect:
- Decrease of flexure amplitude
- Increase of wavelength of deformation

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Inverse calculation of elastic thickness

- Goal:
  - Identify boundaries
  - Characterize crust
  - Find rigidity variations
- Illustration: on synthetic example

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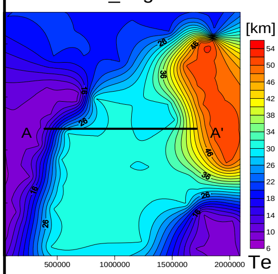
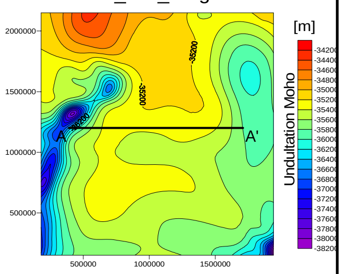
Te grid and Moho for a Te grid

Var_te.grd



Mo_Var_Te.grd



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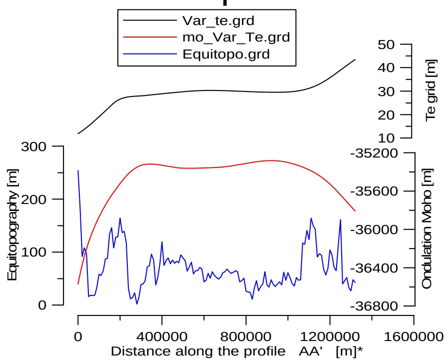
Plots of input files



* the profile is digitized along the small grid of output files (for example: Te.grd)

LEX

To be considered in inversion process of Te

- Choose Te spatial resolution:
  - a,b window size of piecewise constant Te
  - da,db: lateral shift of window for calculations
- Big window: Te is averaged over broader area
- Small window: results are instable
  - Typically: 120-150 km sidelength
- Window shift: can be small. Determines final resolution of Te grid

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To be considered in inversion process of Te

- Ouput of calculation:
- Flexure Moho with piecewise constant Te
- Te grid
- Residual Moho =

  = observed Moho-flexure Moho

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Model consistency check

- Residuals: allow check on internal consistency of model
- Large residuals:
  - Reference depth incorrect
  - Loads incorrect (-> unrecognized buried loads)
  - Failure of flexure model (e.g. subduction zones)
  - Thermal effects
  - Given Moho incorrect

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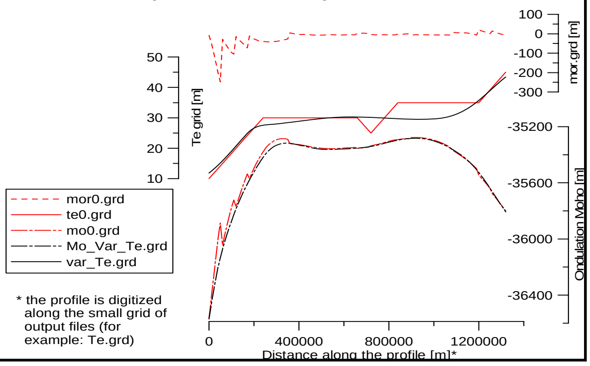
Comparison with input and output files



* the profile is digitized along the small grid of output files (for example: Te.grd)

Example in oceanic area

- Sea mount in South China Sea

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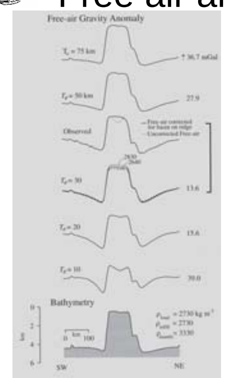
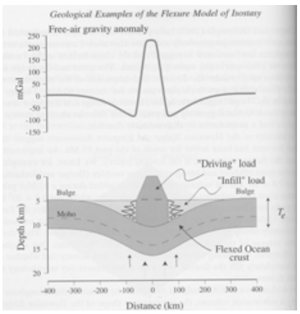
Free air anomaly in Ocean



Watts A.B.



Watts A.B., (2002) Isostasy and Flexure of Lithosphere, (Cambridge University Press)

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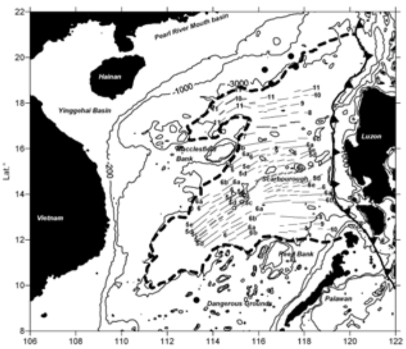
Working area: South China Sea (SCS)



Geological sketch

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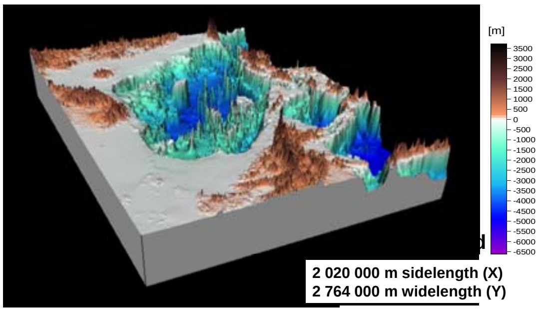
Topography

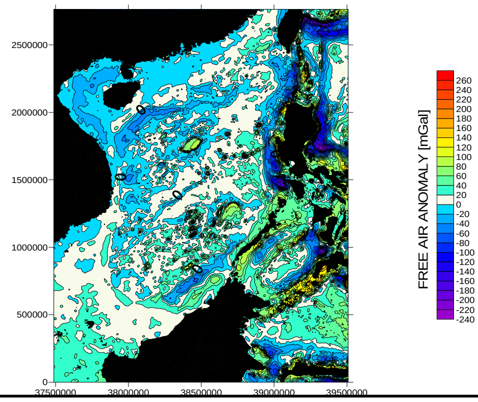


2 020 000 m sidelength (X)
2 764 000 m widelength (Y)

Xmin=37 488 000 m
Xmax=39 508 000 m
Ymin=0 m
Ymax=2 764 000 m

FLEX

Free Air Anomaly



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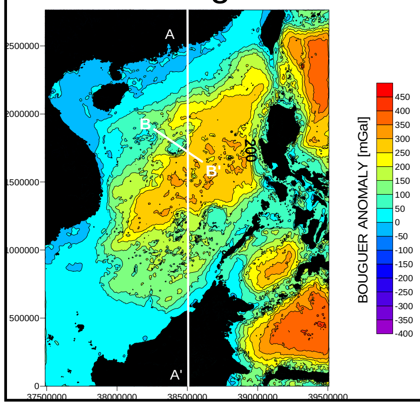
Bouguer-anomalies



Section NS→AA'.bln

Section along the seamounts→BB'.bln

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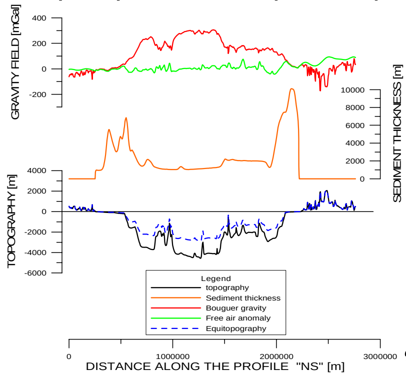
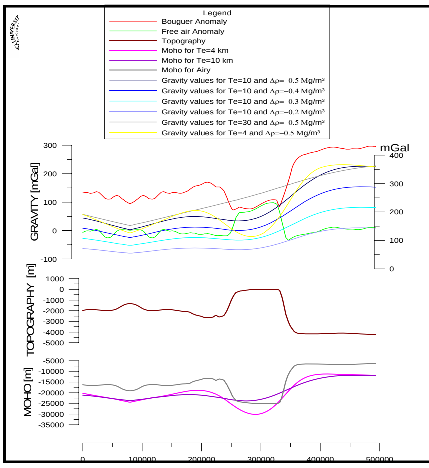
Gravity, topo, load etc.

Example: profile N-S (AA'.bln)



FLEX



Regional and Airy modelling

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Summary

- Introduction to the flexure of lithospheric plate
- Introduction to the necessary quantities fro forward and inverse flexure modeling
- Conclusions:
- Prepared for second day of PC-Lab of Lithoflex course!

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Thank you for your attention!

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