LITHOFLEX WORKSHOP
24-25 JUNE 2008

StatoilHydro Research Centre Rotvoll – Trondheim – Norway

Lithoflex theoretical background- Part I:

Theoretical background for gravity studies

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StatoilHydro

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Course Program

Day 1 Morning- theory (9:00-11:30)
9:00-9.-15: Welcome

9.15-10:15: **Part I: Theoretical background for gravity studies (CB)**
Theoretical background for gravity forward and inverse calculation by Parker approach. Role of the minimum wavelength in inversion. Parameter trade-off. Sensitivity study of parameters. Grace derived gravity field; GOCE satellite

10:15-10.30: Coffee

10:30-11:30: **Part II: Density-depth functions (JE)**
Density-depth functions in general and for sediments. Velocity-depth relation. Compaction models.

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Day 1 Afternoon- practical (12:30-16:00)
12:30-13:15
-Data preparation. Useful grid sampling. Geosoft and Surfer Grid formats (JE)

14:00-…
Areas to be calculated: 1 - West Siberian Basin.
Introduction to grids: sediment, topography, gravity anomaly, Bouguer anomaly, Moho, seismic sections (Vyssotski). Sediment forward calculation. Testing different density-depth functions. Moho forward gravity calculation, Testing of parameters, gravity residual calculation

(2 - Backup example: South China Sea)

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Day 2 Morning- theory (9:00-11:30)
9:00-9:30: **Part III Introduction to isostasy (SW)**

9:30-10:15: **Part IV Isostatic anomalies and basin evolution (JE)**
isostatic anomalies, local isostasy, Pratt model, McKenzie-rifting, Backstripping

10:15-10.30: Coffee

10:30-11:30: **Part V Regional flexure modelling (CB)**
Regional flexure modelling, full plate and broken plate model, Te constant and variable, Forward and inverse flexure calculations, Necessary constraints: crustal thickness and equivalent load, relative importance of internal loads and topographic loads (CB).

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Day 2 Afternoon- practical (12:30-15:00)
equivalent total load calculation. Synthetic topographic generation. Flexure forward calculation. Continue flexure forward calculation, testing role of parameters. Flexure inverse calculation on a synthetic case.

Final Discussion.

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Topics

- Motivation for Lithoflex
- Theoretical background for gravity forward and inverse calculation by Parker approach.
- Main parameters
- Sensitivity study of parameters.
- Grace derived gravity field.
- GOCE satellite.

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Motivation of isostatic-gravity joint study- Lithoflex approach

- For practical interest: detailed knowledge of small area
- But profitable:
  - – knowledge of broader geological context.
  - – knowledge of geological boundaries

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Example boundaries

- Ocean-continent crustal boundary
- Ancient plate boundaries
- Ancient deformation zone
- Mobile belts
- Suture zones
- Rift sequences
- Rifted crust

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Isostasy and gravity in identifying boundaries

- Assumption:
  - Lithospheric rheology: boundaries may produce differences in physical properties
- In particular of our interest:
  - flexural rigidity
  - density variation

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Flexure model

- Deformation of thin elastic plate.
  - "thin": horizontal layers remain parallel
  - Details on Tuesday
- If a vertical density gradient is present: deformation produces measurable gravity signal.

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Vertical density gradients

- Main vertical density gradients in normal conditions:
  - Topography/bathymetry
  - Sediments/basement
  - Crust-Mantle-boundary
- Flexed crust: deformed density boundary layers

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Evaluate gravity of flexed crust

- Direct approach:
  - Evaluate gravity field of surface with strong density gradient
  - Series expansion due to Parker
- Gravity modeling situations:
  - Inverse: determine undulation of boundary, compare with the expected flexure
  - Forward: you have calculated flexure of crust, determine the corresponding gravity signal
- -> examples in the PC-application this afternoon.

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Gravity signal

- Gravity signal of an undulating boundary:

$$FT[\Delta g(\vec{r})] = 2\pi G\, e^{-|\vec{k}|d} \sum_{n=1}^{\infty} \frac{|\vec{k}|^{n-1}}{n!} FT\left[w_d{}^n(\vec{r})\Delta\rho(\vec{r})\right]$$

Gravity field of a layer bounded by upper and lower surface

$$FT[\Delta g(\vec{r})] = 2\pi G e^{-|\vec{k}|d} \sum_{n=1}^{\infty} \frac{|\vec{k}|^{n-1}}{n!} FT\left[(z_U{}^n(\vec{r}) - z_L^n(\vec{r}))\rho(\vec{r})\right]$$

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Parameters entering the gravity field calculations

- Boundary forward modeling depends on:
  - Reference depth d
  - Density contrast
- Boundary inverse modeling depends on:
  - Reference depth d
  - Density contrast
  - Filtering cut-off wavelength Pmin

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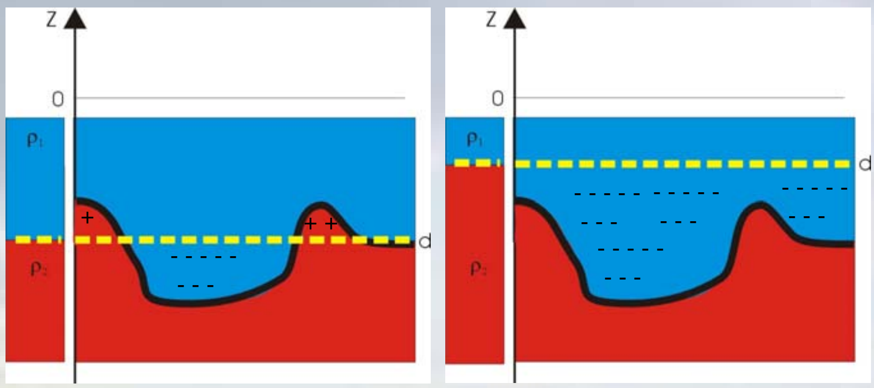
Role of reference depth

forward or inverse calculation



$$\rho_2 > \rho_1$$

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Role of reference depth

gravity inversion

- Effects of reference depth **d**:
  - Shift undulation of boundary vertically
  - Damping of higher wavenumbers
  - For increasing density with depth:
    - boundary above **d**-> positive gravity effect.
    - boundary below **d**-> negative gravity effect
- Notice: **d** not necessarily average depth of layer

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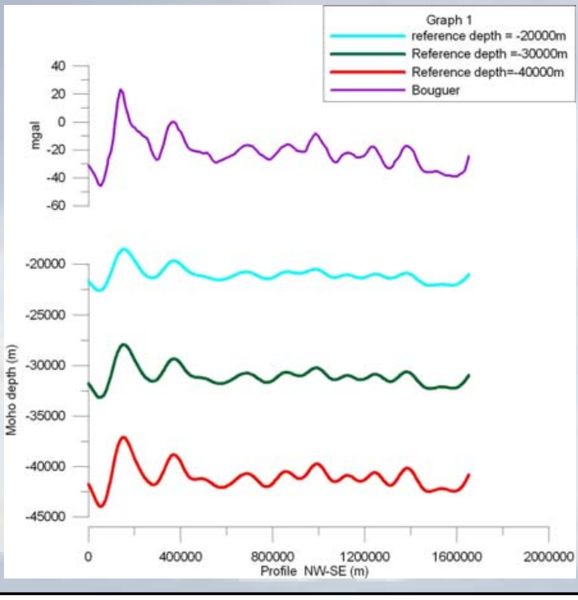
Reference depth

gravity inversion

Graph showing inversion for different reference depths.

Notice Amplitude increase of solution for increasing reference depth.



Amplitude dependence of reference depth

- For inversion process:
- Focus on:
  - wavelength dependency of amplitude of boundary undulation

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Gravity signal and depth of sources

- Earth filter: high spatial frequencies are reduced for increasing source-distance
- What is minimum amplitude placed at Moho level to be detected?

$$FT\left[\Delta g(\vec{r})\right] \approx 2\pi G(\rho_m - \rho_c)\, e^{-\left|\vec{k}\right| d}\, FT\left[w(\vec{r})\right]$$

$$w_k = \Delta g_k \frac{e^{\left|\vec{k}\right| d}}{2\pi G(\rho_m - \rho_c)}$$

With w_k and Δg_k the spectral components of the respective fields for wavenumber k

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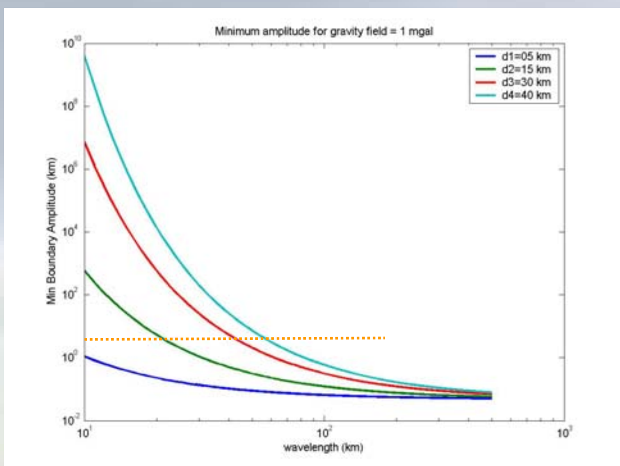
Amplitude estimates

- Gravity resolution=1 mgal, different source-depths, variable wavelengths



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Consequences for inversion process

- Observed field must be frequency limited prior inversion.
- Low pass filter.
  - Cut off wavelength Pmin.
- Next slide:
  - resulting boundary for different choices of Pmin.

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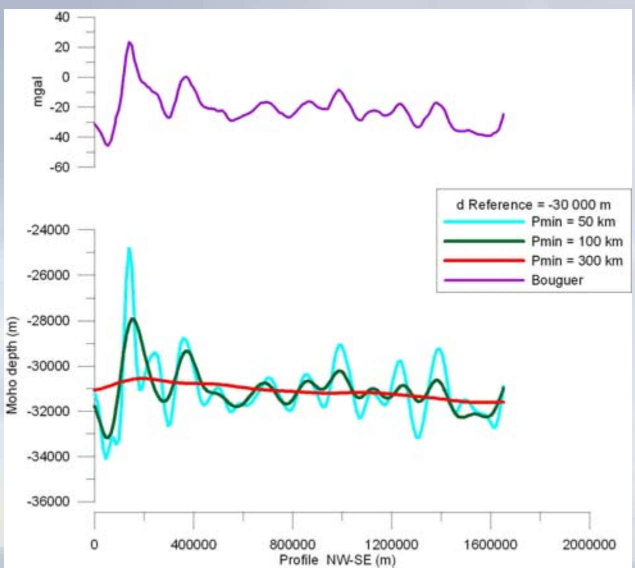
Cut-off wavelength

Graph showing inversion for different Pmin. A small Pmin erroneously projects superficial masses to Moho level.



Density

- Density enters calculation as difference between upper and lower layer
- Solution inversely proportional to density contrast

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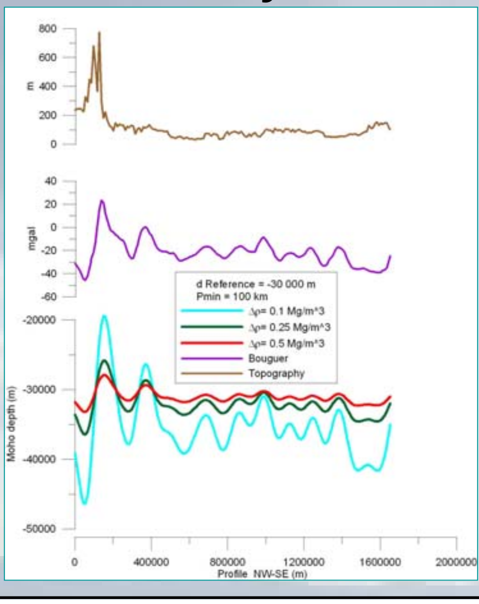
Effect of density variation



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Pitfalls in the inversion

- Gravity inversion: given the model, a solution is found. Only the control with the constraining data or physical model assures solution is correct.
- Operator's responsibility: isolate signal produced by the boundary undulation
  - Forward calculation of known masses: topography/bathymetry, sediments, rifts
  - Frequency filtering of signal

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Pitfalls in gravity inversion

- Low pass filtering not necessarily eliminates superficial masses

  Examples:
  - large sediment basin
- Safely: high frequency-gravity field comes from superficial sources.

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Forward calculation

- Less problematic.
- Only choice of reference boundary and density contrast necessary.
- Observations made for inversion process easily projected into forward calculation process.

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Part B of seminar

- A look on global modern gravity fields
- Recent fields derived from satellite GRACE
- Outlook on the new expected developments from GOCE

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Modern global gravity fields

- GRACE satellite derived fields
  - Pure satellite derived field: N=150
  - Coefficients of higher order: integration of satellite and terrestrial data.
- Max degree and order N: refers to spherical harmonic development

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Spherical harmonic development

$$U(r,\vartheta,\lambda) = \frac{GM}{r}\left(1+\sum_{n=1}^{\infty}\sum_{l=0}^{n}\left(\frac{a}{r}\right)^{n}(C_{nl}\cos l\lambda + S_{nl}\sin l\lambda)P_{nl}(\cos\vartheta)\right)$$

M = Earth mass *G*=Gravitational constant

a= equatorial radius

C_{nl}, S_{nl}: Harmonic coefficients (Stokes coefficients)

$P_{nl}(\cos\vartheta)$: Associated Legendre Polynmials

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Degree and order

- Degree N, oder L
- Spatial resolution:
- The maximum degree *N* of the expansion correlates to the spatial resolution at the Earth surface by

λmin ≈ 40000 km/N

N=360	N=120	N=2160
λmin=110km	*λmin=330km*	*λmin=19km*

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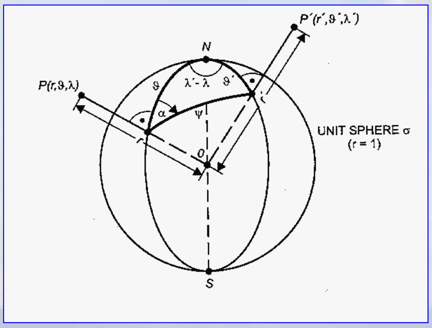
Stokes coefficients

$$C_{n0} = \frac{1}{M} \iiint_{Earth} \left(\frac{r'}{a}\right)^n P_n(\cos\vartheta')\rho(r',\vartheta',\lambda')dV$$

$$\begin{Bmatrix} C_{nl} \\ S_{nl} \end{Bmatrix} = \frac{2}{M}\frac{(n-l)!}{(n+l)!} \iiint_{Earth} \left(\frac{r'}{a}\right)^n P_{nl}(\cos\vartheta') \begin{Bmatrix} \cos l\lambda' \\ \sin l\lambda' \end{Bmatrix} \rho(r',\vartheta',\lambda')dV \quad for\ l \neq 0$$



Satellite method

Input data: *Perturbations of satellite orbits*

Practical solution – **terrestrial gravity data, altimetry**

Principle of the method:

Determination of spherical harmonic coefficients using perturbations of satellites

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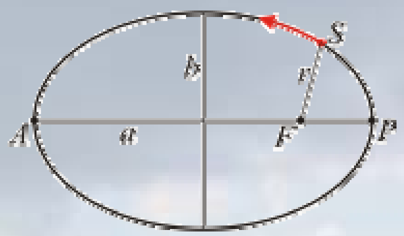
(Michal Sprlak, 2008)

HYPOTHESIS: **Earth is homogeneous sphere, no sun and moon exist, no atmosphere**



NO PERTURBATIONS ON SATELLITE ORBIT!!!

NO CHANGE OF ORBITAL ELEMENTS!!!

Gravitational potential on orbit:

$$\boxed{V(S) = \frac{GM}{r}}$$

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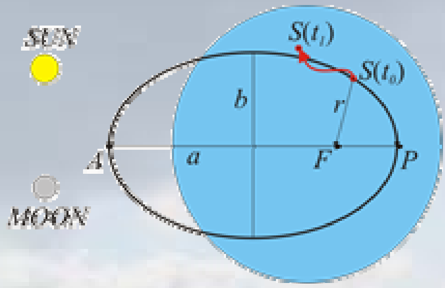
(Michal Sprlak, 2008)

REALITY: **Earth is heterogeneous body, sun, moon and atmosphere exist**



PERTURBATIONS ON SATELLITE ORBIT!!!

CHANGE OF ORBITAL ELEMENTS!!!

Gravitational potential on orbit:

$$V(S)=\frac{GM}{r}\left\{1+\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n}\left[C_{nm}\,R_{nm}(\varphi,\lambda)+S_{nm}\,S_{nm}(\varphi,\lambda)\right]\right\}$$

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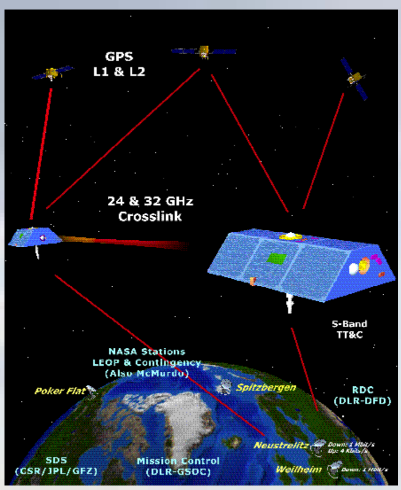
(Michal Sprlak, 2008)

GRACE satellite mission

Gravity Recovery And Climate Experiment



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GRACE satellite mission

- GRACE satellite: launch March 17, 2002.
- H=500 km
- Satellite separation: 200 km along track.
- Distance and rate changes measurement: K-band microwave ranging system.
- Further: GPS receiver, Laser-retro-reflector, star-sensors, three-axis accelerometer

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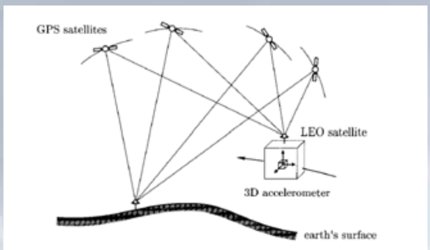
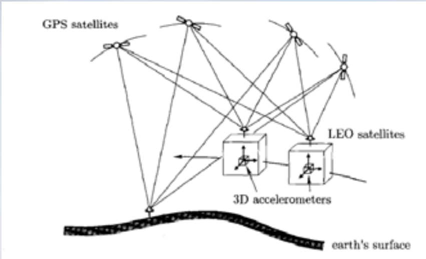
Satellite observations

CHAMP
(Challenging Minisatellite Payload)



GRACE



SST-hl: Satellite-Satellite Tracking in high-low mode
SST-ll: Satellite-Satellite Tracking in low-low mode
LEO: Low Earth orbit satellite

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Current global gravity fields

- EIGEN-GL04C (Förste et al., 2008): N=360
- EGM2008 (Pavlis et al., 2008): N=2160
- EGM96 (Lemoine et al., 1998): N=360

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Calculation methodology of EIGEN-GL04c

- Dynamic approach based on analysis of orbit perturbations of GRACE and LAGEOS satellites n<150
- N>120: Compilation of surface gravity data: satellite altimetry, ship-borne/ airborne gravity over oceans
- Airborne + terrestrial over land: 30'x30' grid

Förste et al., 2008

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Data sources for EIGEN-GL04c

- Terrestrial principal source: NGA (Nat. Geospatial Intelligence Agency, former NIMA) 30'x30' grid
- OCEAN: GFZ Mean Surface height (ERS-1, ERS-2, Topex/Poseidon) minus ECCO Circulation model of oceanic surface.
- Satellite field: 2 years of GRACE data (2002-2005)

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Practical solution:

GLOBAL GRAVITY FIELD MODELS

- pure satellite
- combined

EIGEN-GL04C potential coefficients up to degree and order 360.

360 6378136.3 0.3986004415E+15

2	0	-0.484165371736E-03	0.000000000000E+00	0.35610635E-10	0.00000000E+00
2	1	-0.186987635955E-09	0.119528012031E-08	0.10000000E-29	0.10000000E-29
2	2	0.243914352398E-05	-0.140016683654E-05	0.53739154E-10	0.54353269E-10
3	0	0.957254173792E-06	0.000000000000E+00	0.18094237E-10	0.00000000E+00
3	1	0.202998882184E-05	0.248513158716E-06	0.13965165E-09	0.13645882E-09
3	2	0.904627768605E-06	-0.619025944205E-06	0.10962329E-09	0.11182866E-09
3	3	0.721072657057E-06	0.141435626958E-05	0.95156281E-10	0.93285090E-10
4	0	0.539873863789E-06	0.000000000000E+00	0.10423678E-09	0.00000000E+00
4	1	-0.536321616971E-06	-0.473440265853E-06	0.85674404E-10	0.82408489E-10
4	2	0.350694105785E-06	0.662671572540E-06	0.16000186E-09	0.16390576E-09
4	3	0.990771803829E-06	-0.200928369177E-06	0.84657802E-10	0.82662506E-10
4	4	-0.188560802735E-06	0.308853169333E-06	0.87315359E-10	0.87852819E-10

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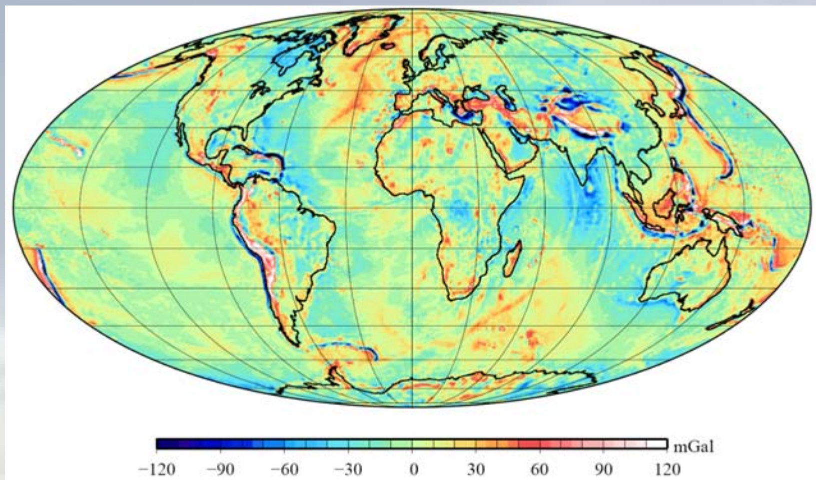
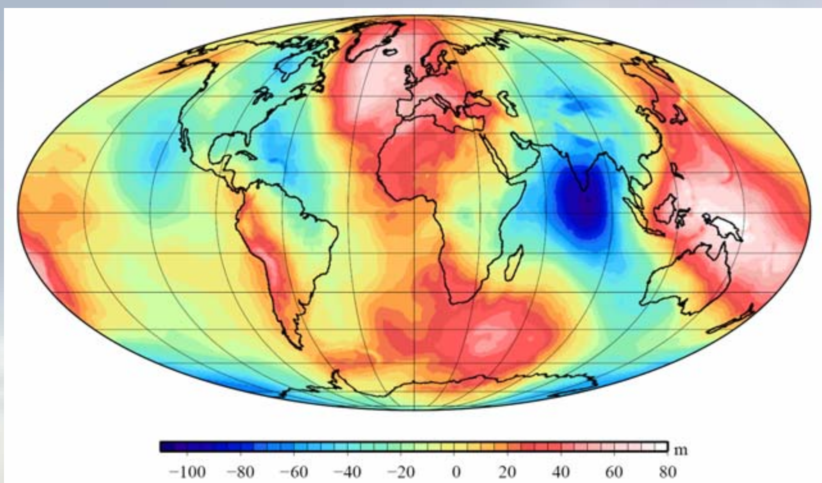
Gravity anomaly - EIGEN-GL04C

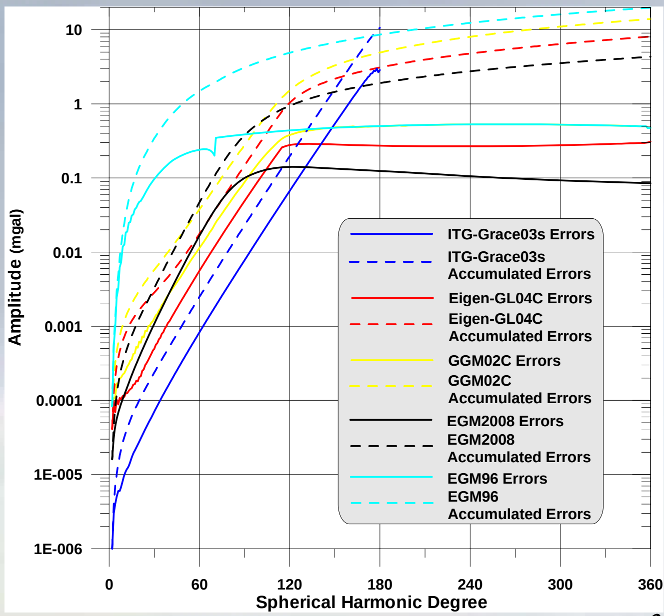


$$\Delta g(r,\vartheta,\lambda)=\frac{GM}{r^2}\left(1+\sum_{n=1}^{N\max}\sum_{l=0}^{n}\left(\frac{a}{r}\right)^n(n-1)(C_{nl}\cos l\lambda+S_{nl}\sin l\lambda)P_{nl}(\cos\vartheta)\right)$$

Geoid undulation - EIGEN-GL04C



$$N(r,\vartheta,\lambda)=\frac{GM}{r\gamma}\left(1+\sum_{n=1}^{N\max}\sum_{l=0}^{n}\left(\frac{a}{r}\right)^n(C_{nl}\cos l\lambda+S_{nl}\sin l\lambda)P_{nl}(\cos\vartheta)\right)$$



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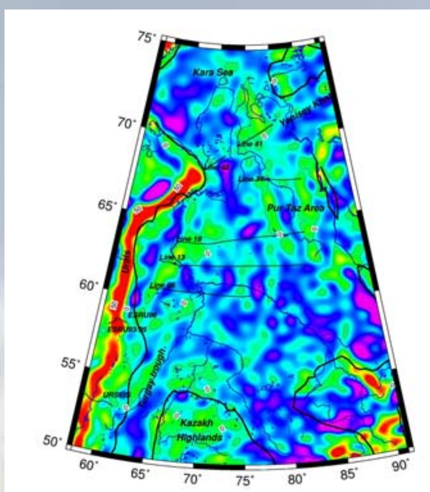
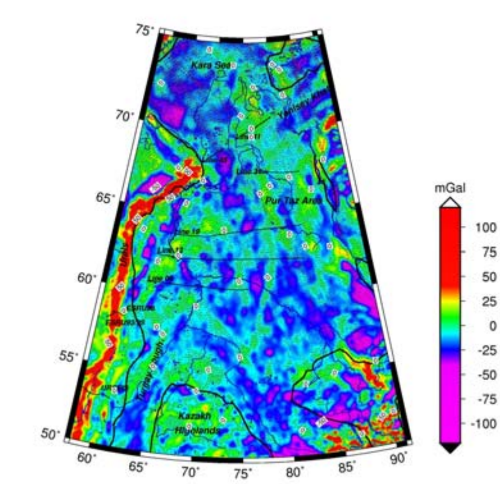
(Shin et al., 2008)

EIGEN-gl04c and EGM08 in WSB



EIGEN-GL04C N=360



EGM08 N=2160

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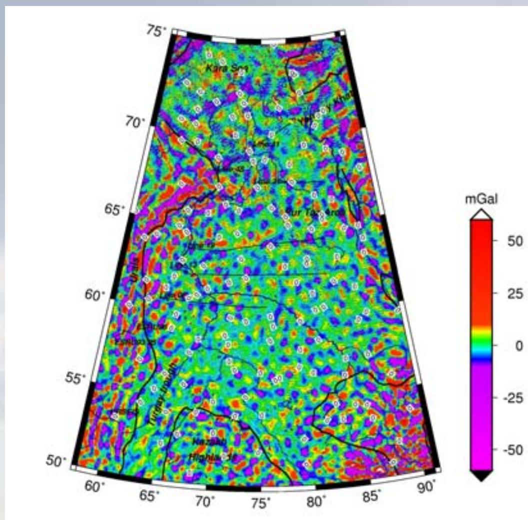
EGM08 minus EIGEN-gl04c



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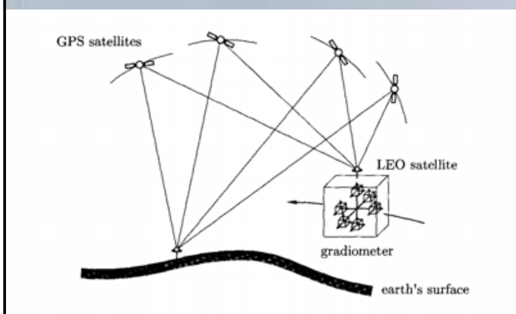
GOCE satellite mission

Gravity-field and steady-state **O**cean **C**irculation **E**xperiment



SST-hl: Satellite-Satellite Tracking in high-low mode.
SGG: Satellite gravity gradiometry

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GOCE characteristics

- Goal: determine stationary gravity field
- Accuracy 1 mgal, Geoid 1-2 cm, 100 km resolution
- Low orbit (250 km), almost polar
- Measurement: Gradiometer and position
- Position: high-low satellites (GPS and GLONASS) and IGS terrestrial permanent stations
- Error in position: cm-level in 3D.

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GOCE characteristics

- Non-gravitational forces corrected from on-board accelerometer measurements
- Satellite only field: up to degree and order n=200-250 (according to reference)
- Gradiometer:
  - 3-axis. 6 capactivie accelerometers
  - Gradient error: $60\, mE / \sqrt{Hz}$
  - 1 Eötvös (E)= 10^{-9} m/s^2/m=10^{-7} Gal/m
  - V_{zz} = vertical gradient on Earth = 3000 E

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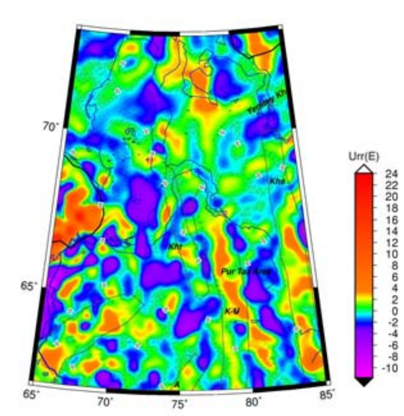
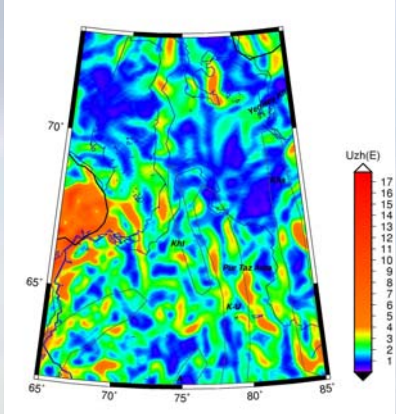
Gradient functions

Vertical gradient of gravity



Horizontal gradient of gravity



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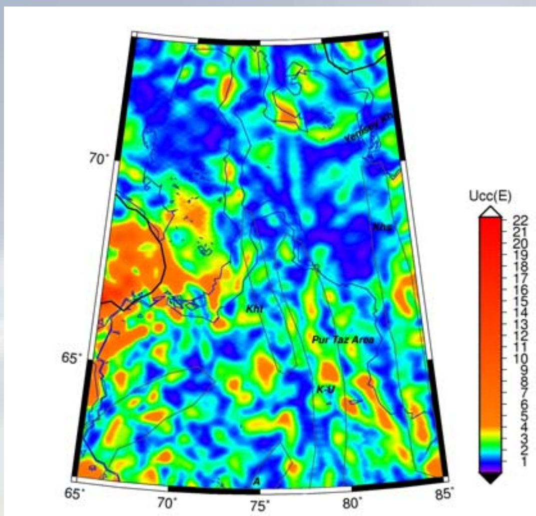
Gradient functions

Curvature



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Summary 1

- Needed for flexure calculations: gravity field of boundary
- Control correctness of crustal internal loads
- Crustal internal load: integration of density anomalies
- Parameters:
  - depth of boundary
  - Density contrast at boundary

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Summary 2

- Global gravity fields: GRACE satellite-terrestrial integration
- New: improvement of harmonics <120.
- Affects global field accuracy
- EGM 2008: examples from West Siberian Basin.
  - Seems to have problems at high latitudes.
  - Striped anomalies. Satellite tracks evident in solution-> interpolation problem?

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Summary 3

- GOCE
  - Launch september 2008
  - improvement of gravity field and geoid. Satellite-only up to N=200-250
  - Tensor components at satellite height
  - examples of tensor components for Pur Taz area, West Siberian Basin.

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Thank you for your attention!

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